

On the Attainability of Fusion Temperatures under High Densities by Impact Shock Waves of Small Solid Particles Accelerated to Hypervelocities*

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In this paper it is shown that temperatures up to 10^8 °K and under densities of the order 1 g/cm^3 are attainable in liquid tritium-deuterium by the impact shock waves of small solid particles accelerated up to velocities of some 10^7 cm/s in heavy particle accelerators.

The high temperatures occur in a focussed particle beam. It is shown that under feasible conditions, the particle beam will generate in the target material a shock wave of the required strength. The particles are charged electrically up to the limit of mechanical breakup and then are accelerated in linear particle accelerators to the required velocities.

In order to cut down losses by Bremsstrahlung radiation, the particles must consist of low Z-value material. The most promising substances in this regard are lithium and beryllium. The "guillotine factor" is of significance at high densities and reduces the Bremsstrahlung losses by a factor of about $1/3$.

The attainable temperatures are high enough to reach the lowest ignition temperatures for thermonuclear reactions.

Apart from the interesting prospect of high temperature, high density experiments, the possibility cannot be excluded to ignite by this method a small fusion explosion of controllable size.

One of the basic difficulties inherent in all devices under study and having as a purpose to achieve a controllable release of fusion energy, is the low operational particle density 10^{15} cm^{-3} . The relatively low densities lead to the requirement of a sufficiently long confinement time in which all the well known instabilities become important.

In a device, uncontrollable however, and known as the H-bomb, this difficulty is avoided by heating the thermonuclear material up to the ignition temperature within a very short time. This is accomplished by a very strong shock wave generated with a fission bomb and which propagates at velocities of the order of some 10^7 cm/s and at densities of the order 1 g/cm^3 . From the well known shock relations, it follows that shock waves strong enough to reach the thermonuclear ignition temperature and which propagate in dense material of the order 1 g/cm^3 are associated with extremely high pressures. From this fact it is clear that all magnetically-driven shock devices can operate at the desired ignition temperatures only at modest densities (we consider a density of $10^{15} \text{ particles/cm}^3$ low compared with a density of $10^{22} \text{ cm}^{-3} \sim 1 \text{ g/cm}^3$).

We would like to show in this paper that there is at least one other possibility of generating a shock wave comparable in strength with the shock wave produced by an exploding fission bomb. This shock wave, however, in contrast to the shock wave of a fission bomb, is of controllably small size.

The basic principle behind the idea is very simple. Suppose we can accelerate a macroscopic solid particle up to a velocity of some 10^7 cm/s . If we then shoot this particle on a target consisting of dense thermonuclear material, temperatures in the range of some 10^7 °K can be achieved. The main purpose of this investigation is to show that macroscopic solid particles can indeed be accelerated to the required velocities of some 10^7 cm/s .

Let us consider a projectile moving at supersonic velocity through a gas or some other compressible fluid. From transsonic gas dynamics it is well known that a projectile, moving through a fluid at supersonic velocity will generate a shock wave. The shock wave can be very strong depending upon the velocity of the projectile and the density and chemical composition of the fluid. If the speed of the projectile, traveling through dense material, is of the order of

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some 10^7 cm/s, the strength of the shock wave is comparable to the strength of a shock wave produced by an exploding fission bomb.

If we accelerate the projectile in some kind of an accelerator up to this speed, we have by-passed the difficulty of heating. In doing this, we have actually performed some kind of heating but it is anisotropic in character and results in a highly anisotropic velocity distribution. The pressure, as a consequence, is also highly anisotropic and has components in one direction only. The projectile seen from a co-moving coordinate system does not change its lokal Maxwellian temperature and for this reason remains cold.

The considered kind of heating may thus be called a „cold heating“ mechanism. The reason for this contradiction in words results from the unclearly defined meaning of what is cold and what is hot. The usual definition, not applicable here, is based on a Maxwellian velocity distribution. In this connection, it should be kept in mind that it is important only to achieve a high speed. This can be done either by „hot“ methods, resulting in a highly isotropic Maxwellian-like velocity distribution, or by „cold“ methods, which in contrast produce a one-dimensional, highly anisotropic velocity distribution. In „cold“ methods, the gas pressure is isotropic, with regard to all spatial directions. In contrast to the „hot“ method, the pressure in the „cold“ method acts in only one direction, the direction parallel to the projectile trajectory. The „hot“ method can operate at high densities only if the confining force is very strong, or the heating power very high and thus the time to keep the material together very short. In an H-bomb, the material is kept together by very strong inertial forces resulting from the short, very high energy release.

In magnetohydrodynamic devices both the heating power release and the confining forces are limited by technical factors. As a result, only relatively low plasma densities are attainable. In the considered „cold“ method in which a projectile is accelerated to a very high speed, the confinement problem is by-passed in all but one spatial direction. As a consequence, in the „cold“ method much higher mass densities are possible. The major difficulty and technical limitation of the „cold“ method arises because confinement is not possible along the trajectories of the accelerated projectiles.

Any kind of solid particles may serve as projectiles. Low Z -value particles consisting of solid lithium or beryllium may favorably serve as projectiles. We will show that the acceleration of the solid particles can be accomplished in electrostatic macroscopic particle accelerators. We will further show that the size of the apparatus increases with increasing particle size. From the condition that the accelerator, because of technical limitations, must be kept below a reasonable maximum length, there results a maximum particle size.

If the particle has reached its final speed, it is shot on the target consisting of a tritium-deuterium mixture. A shock predicted from gas dynamical considerations, however, is possible only if the particle size is larger than the mean-free path. Since the mean-free path increases with the square of the absolute temperature and since large particles require large scale accelerators, it is difficult but not impossible to satisfy this shock condition. The difficulty can be bypassed by using an intense beam of solid particles and confining the beam on the target by electrostatic lenses. In this case the condition for the onset of a shock can be satisfied by the collective particle behavior.

If such a beam is concentrated on the target it will be able to produce a high temperature, high density shock wave thus creating conditions comparable to those presumably present in stellar interiors.

Using as target material liquid or solid tritium-deuterium and a particle material of low Z value such as lithium, the ignition temperature of the T-D fusion reaction can be reached. It may be therefore not completely impossible to ignite a small thermonuclear explosion of controllable size.

1. Macroscopic Particle Accelerators

In order to make a feasibility study of the proposed method we have first to answer the most important question, that is, how to accelerate macroscopic particles to the required velocities in the range of some 10^7 cm/s. It has been shown in a very interesting paper by SHELTON, HENDRICKS and WUERKER¹ that micron size particles can be accelerated in electrostatic accelerators to very high velocities. The application these authors had in mind

¹ H. SHELTON, C. D. HENDRICKS, and R. F. WUERKER, *J. Appl. Phys.* **31**, 1243 [1960].

was the study of the impact damage on solid targets in the 10^6 cm/s velocity range. Heavy particle accelerators have been also proposed for space propulsion systems².

In order to determine the efficiency of such heavy particle accelerators, we start with some basic considerations. (We are using electrostatic units for all derivations.) The force acting on the particle possessing a charge q and in an electric field E is given by:

$$F = qE. \quad (1.1)$$

It is assumed that the particle is positively charged to prevent field emission effects. The maximum attainable charge is then determined by the mechanical strength of the material of which the particle consists. This means that the electric field at the surface of the particle resulting from the charge q cannot exceed a certain maximum value E_0 . This maximum value E_0 is determined from equating the tensile strength of the material σ_0 with the "electric pressure" $E_0^2/8\pi$,

$$E_0 = \sqrt{8\pi\sigma_0}. \quad (1.2)$$

Inserting into (1.2) the well-known values³ of σ_0 for lithium and beryllium we get⁴

$$\text{lithium: } E_0 = 2.1 \times 10^5 (\text{esu}),$$

$$\text{beryllium: } E_0 = 3.3 \times 10^5 (\text{esu}).$$

The maximum amount by which a sphere of radius r can be electrically charged is thus given by:

$$q = r^2 E_0. \quad (1.3)$$

If the density of the sphere is ρ , the acceleration calculated from (1.1) and (1.3) is given by:

$$a = (3/4\pi) E_0 E / \rho r. \quad (1.4)$$

If the particle shall be accelerated up to a velocity v the length l of the accelerator is

$$l = v^2/2a = (2\pi/3) \rho v^2 r / E_0 E. \quad (1.5)$$

In the next chapter it is shown that the particle must have at least a velocity of 7.2×10^7 cm/s to cause a shockwave temperature of 3×10^7 °K, the ignition temperature for the T-D reaction.

The density ρ is for lithium 0.534 g/cm³ and for beryllium 1.84 g/cm³. To obtain some definite numerical values, it is assumed that the accelerating

electric field is equal to 10^5 volts/cm = 3.3×10^2 esu. Substituting this value, together with the value E_0 and ρ for lithium and beryllium and the required value of v into (1.5) we get:

$$\text{lithium: } l = 8.3 \times 10^7 r \quad (\text{cm}),$$

$$\text{beryllium: } l = 1.8 \times 10^8 r \quad (\text{cm}),$$

In Figure 1, the length l of the accelerator size is given in dependence of the particle radius r .

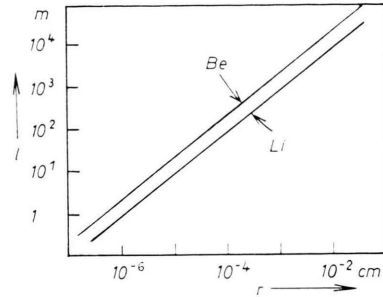


Fig. 1. Linear accelerator dimension as a function of particle size to achieve a velocity of 7.2×10^7 cm/s, plotted for lithium and beryllium particles.

If we demand that the maximum achievable size of the device should not exceed one kilometer, it follows that the maximum feasible particle size has a diameter of 2.4×10^{-3} cm for lithium and 1.1×10^{-3} cm for beryllium. The total voltage along the particle trajectory is given by:

$$V = El \quad (1.6)$$

which, according to (1.5) can be expressed as follows:

$$V(r) = (2\pi/3) \rho v^2 r / E_0. \quad (1.7)$$

In a linear accelerator of the COCKCROFT WALTON or VAN DE GRAFF type, the highest possible voltages are of the order of 10^6 volts. Inserting this value on the left hand side of (1.7) and solving for r we obtain a value for the maximum particle size which can be accelerated to the required velocity in accelerators of this type. Using the values for lithium and beryllium, we get a particle diameter for lithium of 2.4×10^{-6} cm and for beryllium of 1.1×10^{-6} cm.

For larger particle sizes, linear accelerators of the waveguide type must be used. If the separation distance of the drift tubes in a linear accelerator is

² R. H. GODDARD, Methods and Means for Producing Electrified Jets of Gas, U.S. Patent 1.363.037, December 21, 1920.

³ These values may be considered conservative since the

value σ_0 for the material strength of single crystals can be larger by a factor 10^2 .

⁴ Solid materials with reasonable high tensile strength and low Z value are lithium and beryllium.

of the order of 10 cm, the waveguide frequency must be in the range of 10^6 cycles/sec. This frequency is low if compared with frequencies of linear accelerators for atomic particles. For this reason powerful radiofrequency equipment is required.

We would like to discuss briefly the possible advantages of cyclic accelerators which make use of magnetic fields. From the condition that the centrifugal force is balanced by the LORENTZ force, we obtain:

$$(4\pi/3) \varrho r^3 v/R = (q/c) H. \quad (1.8)$$

In (1.8), R is the curvature radius of the particle orbit moving in a magnetic field H . q is given by equation (1.3). Substituting (1.3) into (1.8) and solving for r leads to

$$r = (3/4\pi) E_0 R H / v \varrho c. \quad (1.9)$$

Eliminating r from (1.9) and (1.5) we can compare the size of the cyclic to the size of the linear accelerator:

$$R/l = 2(E/H)(c/v). \quad (1.10)$$

With superconducting coils, magnetic fields of 10^5 Gauss are feasible. We thus obtain from (1.10)

$$2\pi R/l \simeq 30. \quad (1.11)$$

As a result, cyclic accelerators have no advantage since the required circumference is larger by a factor 30 compared with the dimension of a linear accelerator.

Next we have to discuss the space-charge limitation in the considered electrostatic accelerator. This question is of importance in connection with the limitations for the beam intensity.

The limitation resulting from space-charge effects is given by the SCHOTTKY-LANGMUIR space-charge equation:

$$j = (2q/m)^{1/2} V^{3/2} / 9\pi x^2 \quad (1.12)$$

where j is the electric current density. The applied voltage shall act along the distance x which is equal to the separation distance between the drift tubes. We define as n the number density of the solid particles.

We have therefore:

$$j = n q v. \quad (1.13)$$

Expressing q by (1.3) and putting $V = Ex$ we get from (1.12) for the number density n :

$$n = (3\pi v)^{-1} (6\pi E_0 \varrho x)^{-1/2} E^{3/2} r^{-5/2}. \quad (1.14)$$

In order to obtain a numerical value we assume a separation distance $x = 10$ cm and obtain:

$$\text{lithium: } n = 0.61 \times 10^{-8} r^{-5/2} \quad (\text{cm}^{-3}),$$

$$\text{beryllium: } n = 2.4 \times 10^{-9} r^{-5/2} \quad (\text{cm}^{-3}).$$

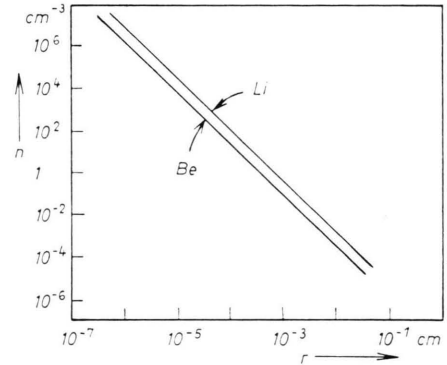


Fig. 2. Maximum number density of solid particles as a function of the particle radius.

In Figure 2 the number densities are given as a function of the particle radius.

Two other important conclusions can be obtained immediately from (1.14). First, the average distance between the solid particles is:

$$d = n^{-1/3} = (3\pi v)^{1/3} (6\pi E_0 \varrho x)^{1/6} E^{1/2} r^{5/6}. \quad (1.15)$$

Inserting the same numerical values as before results in:

$$\text{lithium: } d = 5.4 \times 10^2 r^{5/6} \quad (\text{cm}),$$

$$\text{beryllium: } d = 7.4 \times 10^2 r^{5/6} \quad (\text{cm}).$$

These values for the separation distances are given in Figure 3 as a function of the particle size r .

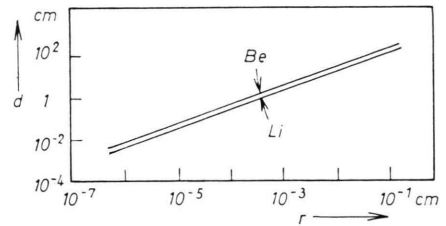


Fig. 3. Average distance between the solid particles as a function of r .

Second, the average number of atoms per cm^{-3} within the beam is:

$$\begin{aligned} N_0 &= (L \varrho / A) (4\pi/3) r^3 n \\ &= (4/9) (6\pi)^{-1/2} (L/A) (\varrho/E_0)^{1/2} E^{3/2} x^{-1/2} v^{-1} r^{1/2} \end{aligned} \quad (1.16)$$

where L is Avogadro's number and A the atomic weight. Inserting numerical values we obtain:

$$\text{Li}^6: N_0 = 1.3 \times 10^{15} r^{1/2},$$

$$\text{Be}^9: N_0 = 1.2 \times 10^{15} r^{1/2}.$$

The atomic number densities are plotted in Figure 4.

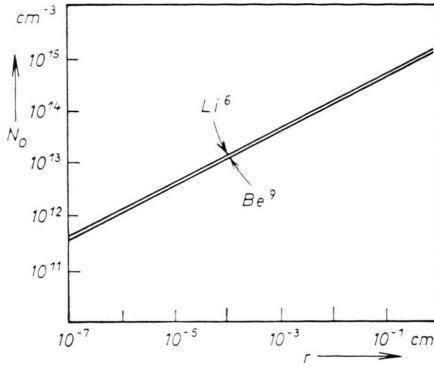


Fig. 4. Maximum number density N_0 of the beam in dependence on r .

The charging of the particles can be accomplished by bringing the particles into contact with a highly charged surface. In order for the particles to be charged they are projected onto the tip of a needle of the kind used in the field-electron microscope.

It is possible to make the diameter of the tip as small as 10^{-3} cm. It is assumed that the shape of this tip is a half sphere. If the sphere of radius r representing the particle is brought into contact with a sphere of radius R representing the needle tip, and if the sphere with radius $R > r$ is at a potential V , the particle will be charged by an amount q given by:

$$q = (\pi^2/6) R r^2 V / (r + R)^2. \quad (1.17)$$

Substituting for q the expression given by (1.3) we get from (1.17):

$$V = (6/\pi^2) E_0 (r + R)^2 / R. \quad (1.18)$$

If $r \ll R$, we may also write:

$$V = (6/\pi^2) E_0 R. \quad (1.19)$$

Putting $R = 10^{-3}$ cm we obtain the potential of the needle necessary to charge the lithium and beryllium particles up to the maximum possible value:

$$\text{lithium: } V = 3.8 \times 10^4 \text{ volts,}$$

$$\text{beryllium: } V = 6.0 \times 10^4 \text{ volts.}$$

In order to obtain the required high current density, many particles must be charged per unit time. This

can be done as follows: First, by electron bombardment the particles are given a slight negative charge. By this negative charge they are attracted towards the tip of the needle, there charged positively and then emitted into the accelerator. Second, to obtain a high particle current density we may operate with a very large number of needles (as indicated by Figure 5) arranged on a concave spherical surface so that the charged particles are automatically focussed into the beam entering the accelerator.

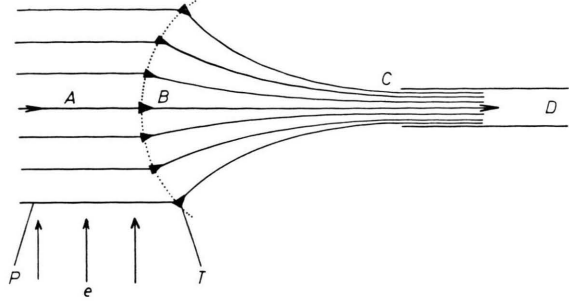


Fig. 5. The charging of the particle beam. At A the particles P obtain by electron bombardment e a small negative charge and at B are attracted towards the needle tips T. The needles are arranged on a concave spherical surface. At C the particles are automatically confined into the beam which enters into the accelerator D.

2. The Dependence of the Temperature and Density Behind the Impact Shock Wave Upon the Impact Velocity and Target Density

If a projectile at transsonic velocity is moving through a compressible medium, three regions must be distinguished. (Figure 6.)

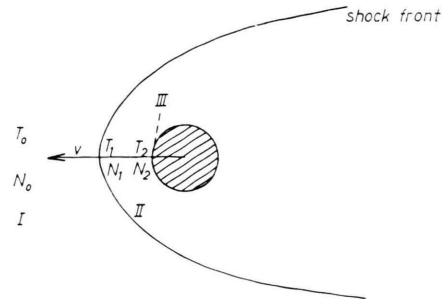


Fig. 6. The different regions of importance in calculating the temperature and density caused by the impact of a particle at transsonic velocities.

First there is the region I still unaffected by the particle impact, its density and temperature given by N_0 and T_0 . The second region in which the density

and temperature rises to N_1 and T_1 is divided from the first region by a shockfront located somewhat between region I and the particle. Finally, if the gas comes to rest at the surface of the projectile the density and temperature rises to N_2 and T_2 . We will see that the significant rise takes place in going from region I to region II.

The transition from region I to region II is determined by the well known shock relations which simplify for strong shocks^{5, 6}:

$$\frac{T_1}{T_0} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_0^2, \quad (2.1)$$

$$\frac{N_1}{N_0} = \frac{\gamma+1}{\gamma-1}, \quad (2.2)$$

$$M_1^2 = \frac{\gamma-1}{2\gamma} \quad (2.3)$$

where $\gamma = c_p/c_v$ is the ratio of specific heats. Because of the high temperatures it is justified to assume $\gamma = 5/3$. M_0 and M_1 are the MACH-numbers in front of and behind the shock.

From this follows:

$$M_0^2 = v^2/c_0^2 = A v^2/\gamma R T_0, \quad (2.4)$$

In (2.4) R is the gas constant $R = 8.314 \times 10^7$ erg/°K.

From (2.1), (2.2) and (2.3), assuming $A = 2.5$ for the T-D target, we thus obtain:

$$T_1 = 3 A v^2/16 R = 5.4 \times 10^{-9} v^2 (\text{°K}), \quad (2.5)$$

$$N_1 = 4 N_0, \quad M_1^2 = 1/5. \quad (2.6), (2.7)$$

If the gas behind the shock front, already heated up to a high temperature, comes to rest on the particle surface the temperature and density rise to still higher values given by:

$$T_2/T_1 = \left(1 + \frac{\gamma-1}{2} M_1^2\right) = 1.07, \quad (2.8)$$

$$N_2/N_1 = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{1/(\gamma-1)} = 1.1. \quad (2.9)$$

Combining (2.5), (2.6) with (2.8), (2.9) leads finally to the maximum temperature and density attainable by the impact:

$$T_2 = 6.0 \times 10^{-9} v^2 (\text{°K}), \quad (2.10)$$

$$N_2 = 0.92 \times 10^{23} \text{ (cm}^{-3}\text{)}. \quad (2.11)$$

In (2.11) we assumed a target density of $N_0 = 2.1 \times 10^{22} \text{ (cm}^{-3}\text{)}$ valid for liquid hydrogen. In

Figure 7, the maximum attainable temperature is given as a function of the particle velocity v .

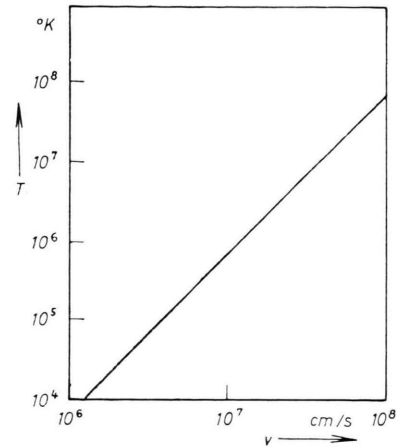


Fig. 7. Maximum temperature attainable by the impact of particles accelerated to hypervelocities.

3. The Condition for the Onset of a Shock

In order to apply the gasdynamical calculation for the maximum attainable temperature it is necessary that the projectile size is larger than or at least of the same order of magnitude as the mean-free path. The mean-free path in a fully ionized hydrogen plasma is given by:

$$\lambda = 1.2 \times 10^4 T^2/N. \quad (3.1)$$

Substituting into (3.1) $T = 3 \times 10^7$ °K and $N = 0.92 \times 10^{23} \text{ cm}^{-3}$ we obtain $\lambda = 1.2 \times 10^{-4} \text{ cm}$.

In order to generate a shock-wave the particles must therefore possess a diameter of at least 10^{-4} cm . The length of the accelerator necessary to bring a particle of this size up to the required velocity of $7.2 \times 10^7 \text{ cm/s}$ is at least 80 m.

If, instead of one particle, a particle beam of sufficient intensity is used, a shock wave may be generated with particles considerably smaller in size than the mean-free path.

It should be possible to concentrate the beam on the target by an electrostatic lens system, and, in order to avoid space charge limitations, to neutralize the beam by electron injection between the lens and the focus into which the particle beam will converge. From electron optics it is well-known that the focal

⁵ Since the required temperature T_1 must be very high it is permitted to neglect the modifications of the shock relations by dissociation and ionisation.

⁶ L. LANDAU and E. LIFSHITZ, Fluid Mechanics, Pergamon Press, New York 1959, p. 317 and 331.

length of an electrostatic lens does not depend upon the q/m value. For this reason a spread in the q/m value will not result in a spread of the focal length. The size of the focus is thus determined by the thermal spread only. This thermal spread is very small due to the high particle mass.

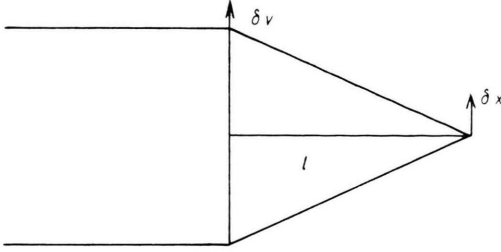


Fig. 8. To the calculation of the thermal beam spread.

We estimate this spread using the notation of Figure 8. If the velocity perpendicular to the beam is given by δv the size of the focus δx will be of the order:

$$\delta x = l v_{\perp} / v. \quad (3.2)$$

The thermal velocity spread v_{\perp} can be estimated from

$$m v_{\perp}^2 \simeq k T_0, \quad (3.3)$$

$m = (4\pi/3)\rho r^3$ is the mass and T_0 the injection temperature of the particles. From (3.3) we thus obtain:

$$v_{\perp} = \delta v = (k T_0 / \pi \rho)^{1/2} r^{-3/2}. \quad (3.4)$$

We assume an injection temperature of $T_0 = 10^3$ °K, with the results:

$$\text{lithium: } \delta v = 2.9 \times 10^{-7} r^{-3/2} \quad (\text{cm/s}),$$

$$\text{beryllium: } \delta v = 1.6 \times 10^{-7} r^{-3/2} \quad (\text{cm/s}).$$

Combining this result with (3.2) and putting $v = 7.2 \times 10^7$ cm/s we obtain:

$$\text{lithium: } \delta x / l = 4.1 \times 10^{-15} r^{-3/2},$$

$$\text{beryllium: } \delta x / l = 2.2 \times 10^{-15} r^{-3/2}. \quad (3.5)$$

In Figure 9, a plot is made of the dependence of $\delta x/l$ upon the particle radius.

From the calculated focal size one can obtain an estimate of the maximum particle densities attainable in a converging neutralized heavy particle beam. If the initial beam radius is R , the factor by which the initial density increases is then clearly given by:

$$N_{\max} / N_0 = (R / \delta x)^2 \quad (3.6)$$

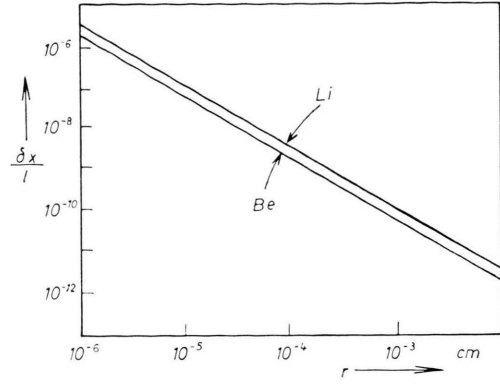


Fig. 9. Minimum focal size resulting from thermal beam spread as a function of particle radius.

Or:

$$\text{lithium: } N_{\max} / N_0 = 5.9 \times 10^{28} (R/l)^2 \cdot r^3,$$

$$\text{beryllium: } N_{\max} / N_0 = 2.1 \times 10^{29} (R/l)^2 \cdot r^3. \quad (3.7)$$

We may combine (3.7) with (1.16) and obtain:

$$\text{lithium: } N_{\max} = 0.8 \times 10^{44} (R/l)^2 r^{7/2} \quad (\text{cm}^{-3}),$$

$$\text{beryllium: } N_{\max} = 2.6 \times 10^{44} (R/l)^2 r^{7/2} \quad (\text{cm}^{-3}).$$

It should be emphasized that the estimate (3.8) is correct only as long as the calculated number density N_{\max} is smaller than the number density of atoms present in the solid particles, which for lithium is equal to $N(\text{Li}^6) = 5.36 \times 10^{22} \text{ (cm}^{-3})$ and for beryllium $N(\text{Be}^9) = 1.23 \times 10^{22} \text{ (cm}^{-3})$.

If the number density calculated from (3.8) is equal to or larger than the number density of atoms in the solid particles, the beam is completely condensed. Assuming a value of $R/l = 10^{-1}$ we may use equation (3.8) to obtain the smallest particle size which will lead to complete condensation of the beam. The result is that the minimum required particle radius for lithium and beryllium is of the order 3×10^{-6} cm. It should be pointed out that particles of this size can already be accelerated up to the required velocity of 7.2×10^7 (cm/s) in COCKROFT WALTON or VAN DE GRAFF type accelerators. Accelerators of this type possess a considerable advantage compared with linear wave guide accelerators and, therefore, will lead to considerable simplification of the described device.

If complete condensation has been achieved, the only necessary condition to develop a shock is that the size of the focus be larger than the mean-free path. From the dependence of the focal size on the particle radius, assuming a focal length of 10^3 cm,

we obtain for particles with $r = 3 \times 10^{-6}$ cm a thermal spread in the focus of $\delta x = 10^{-3}$ cm. This diameter is larger by one order of magnitude than the mean-free path at fusion temperatures.

The maximum size R of the focus is calculated from the condition

$$N_0 R^2 = N_{\max} R_1^2$$

respectively

$$R_1 = \sqrt{N_0/N_{\max}} R. \quad (3.9)$$

If complete beam condensation is achieved one must substitute for N_{\max} the values for solid lithium and beryllium and for N_0 the values given by equation (2.16). From this follows:

$$R_1 \simeq 10^{-4} r^{1/4} R \quad (\text{cm}) \quad r > 3 \times 10^{-6} \quad (\text{cm}) \quad (3.10)$$

(approximately valid for both substances.)

For the minimum particle radius required to permit beam condensation, $r = 3 \times 10^{-6}$ cm, equation (3.8) leads to

$$R_1 \simeq 10^{-5} R.$$

For an initial beam radius of 10^2 cm the diameter of the focus is thus $R_1 \sim 10^{-3}$ cm, which is larger by a factor of 10^1 than the mean free path at fusion temperatures.

The beam under this condition will generate in the target a shockwave with an extension comparable with this size of 10^{-3} cm.

4. Some Energy Considerations and Their Consequences

We will now show that even without complete beam condensation a shock wave-like structure may develop depending upon the energy supply of the beam to the target.

Since the theory for the onset of shock waves generated by the impact of a large number of smaller than mean-free path size particles is very difficult, it is preferable to apply energy considerations.

The maximum beam energy flux, Φ_{\max} , is clearly given by:

$$\Phi_{\max} = \frac{1}{2} N N_{\max} v^3 \quad (\text{erg/cm}^2 \text{ s}). \quad (4.1)$$

In (4.1) M is the atomic mass of the beam material, N_{\max} the maximum atomic density reached in the focus and v the critical velocity of 7.2×10^7 (cm/s).

If the range of the beam particles is given by λ_r , the energy dissipated per unit volume and time is given by:

$$Q_B = \Phi_{\max}/\lambda_r \\ = \frac{1}{2} M N_{\max} v^3/\lambda_r \quad (\text{erg/cm}^3 \text{ s}). \quad (4.2)$$

This dissipation of kinetic beam energy is to be compared with the energy loss by radiation⁷,

$$Q_r = 1.42 \times 10^{-27} T^{1/2} g \cdot \sum_n w_n (N_e N_i Z_n). \quad (4.3)$$

In (4.3) w_n is the relative weight factor of the different elements present in the radiating substance. For an equal hydrogen-lithium mixture we get:

$$Q_r = 1.42 \times 10^{-26} T^{1/2} N^2 g, \quad (4.3a)$$

g is the "guillotine factor". For high densities of the order 1 g/cm^3 g is given approximately by:

$$g = 0.25 \times \varrho^{-0.25} \quad (4.4)$$

where ϱ is the mass density. For $\varrho = 0.35$ (Li-T-D mixture) we get $g \simeq 0.32$. With $T = 3 \times 10^7$ °K and $N = 2.1 \times 10^{22} \text{ cm}^{-3}$ (particle density in liquid hydrogen) it thus follows from (4.3a) that

$$Q_r = 1 \times 10^{22} \text{ erg/cm}^3 \text{ s}.$$

To calculate the dissipated beam energy, we have to know λ_r . The drag force F , acting on the particle is given by:

$$F \simeq \pi \varrho_t v^2 r^2 \quad (4.5)$$

where ϱ_t is the density of the target material and ϱ_t (T-D) = 0.175 g/cm^3 . From (4.5) we obtain the deceleration a of the particle:

$$a = (3/4) (\varrho_t/\varrho_p) v^2 r^{-1} \quad (4.6)$$

where ϱ_p is the density of the particle material. The range λ_r of the particle is then given by:

$$\lambda_r = v^2/2a = (4/3) (\varrho_p/\varrho_t) \times r. \quad (4.7)$$

Since $\varrho_p \sim \varrho_t$, it follows that the range $\lambda_r \sim r$. With this result in mind we therefore get from (4.2):

$$Q_B \simeq M N_{\max} v^3/r. \quad (4.8)$$

Inserting the value for N_{\max} given by (3.8), putting for $M \simeq 10^{-23} \text{ g}$ the mass of a lithium atom and assuming $R/l \simeq 10^{-1}$ results in:

$$Q_B \simeq 3 \times 10^{42} r^{5/2} \quad (\text{erg/cm}^3 \text{ s}). \quad (4.9)$$

For a particle radius larger than 3×10^{-6} cm complete beam condensation is possible. Inserting $r = 3 \times 10^{-6}$ into (4.9) yields the highest possible value for the supplied energy per unit volume and unit time:

$$Q_B = 5 \times 10^{28} \quad (\text{erg/cm}^3 \text{ s}).$$

This value is considerably larger than the value for the Bremsstrahlung radiation. From the condition that

$$Q_B \geq Q_r \quad (4.10)$$

⁷ G. GAMOW and C. CRITCHFIELD, Atomic Nucleus and Nuclear Energy Sources, Clarendon Press, Oxford 1947.

it follows that

$$N_{\max} \geq 3 \times 10^{21} r. \quad (4.11)$$

Assume for instance particles of the size $r = 10^{-6}$ cm. It then follows $N_{\max} > 10^{15} \text{ cm}^{-3}$. This means that if $r = 10^{-6}$ cm and $N_{\max} = 10^{15} \text{ cm}^{-3}$ the energy supplied by the beam to the target is of the same order of magnitude as the Bremsstrahlung losses. The beam in this case has an initial particle density of 10^{11} cm^{-3} and is confined in its diameter by a factor of 10^2 , thus increasing the particle density by a factor 10^4 . Inserting these values into equation (3.9) we obtain as the maximum size of the beam focus:

$$R_1 \simeq 10^{-2} R.$$

A beam must be confined at least by this amount to make shock-wave generation possible. Shock-wave generation will begin to become effective if the initial beam radius of 10^2 cm is focussed to a radius less than 1 cm.

The presence of lithium in the target material will increase the radiation losses. The Bremsstrahlung increases rapidly with increasing Z . But at the given high densities the „guillotine factor“ becomes important. Taking both effects — the increase in Bremsstrahlung by a higher Z value and the „guillotine factor“ — into account the radiation losses are increased by a factor 3.2. The thermonuclear reaction rate at the ignition temperature for the T-D reaction is a very steep function of the temperature so that an increase on the radiation losses by a factor 3.2 will not result in a drastic increase of the ignition temperature.

5. Conclusion

It has been shown that temperatures up to the ignition temperature of the T-D reaction at 3×10^7 °K are attainable by accelerating microscopic solid particles to hypervelocities of some 10^7 (cm/s). The required accelerator size increases in proportion to the particle size. But even with COCKROFT WALTON or VAN DE GRAFF type accelerators particles of the order of 10^{-6} cm can be accelerated to the required velocities. A beam consisting of such particles can be focussed by an electrostatic lens on a target creating immense power densities. As a result, a shock wave of very high strength is generated in the target material.

The energy dissipation in this shock wave can be made considerably higher than the Bremsstrahlung losses. Fusion temperatures are achieved in the target in a very small volume at densities of condensed matter.

The estimates made in this paper indicate that it may be possible to ignite by the described method a thermonuclear reaction.

Apart from the prospect of igniting a fusion reaction the described device may be of interest for high temperature experiments at high densities which are not attainable in other proposed plasma devices.

Note added in proof: This paper is based on an unpublished report of Case Institute of Technology Plasma Research Program, Technical Report No. A-21, June 1963, and has been presented at the annual meeting of the Plasma Physics Division of the American Physical Society in San Diego Nov. 6–9, 1963.